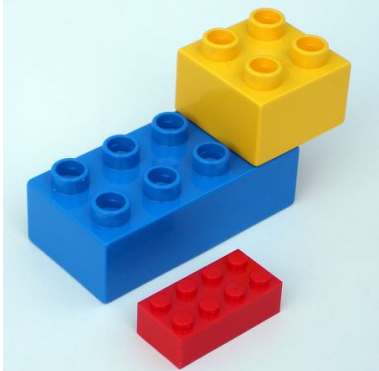


Introduction to Local Estimation



x

→

y₁

→

y₂

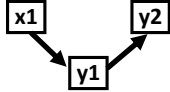
$\text{lm}(y_2 \sim x + y_1)$
 $\text{lm}(y_1 \sim x)$

Overview

1. Covariance vs. piecewise SEM
2. Assessing Model Fit: Tests of directed separation
3. Introduction to *piecewiseSEM*
4. Further adventures into *piecewiseSEM*

Covariance Based SEM

Hypothesized Model



↓

Parameter Estimates

Observed Covariance Matrix

$$S = \begin{Bmatrix} 1.3 & & \\ .24 & .41 & \\ .01 & 9.7 & 12.3 \end{Bmatrix}$$

estimation
(e.g., maximum likelihood)

←

Evaluate Model Fit

→

compare

→

$$\hat{\Sigma} = \begin{Bmatrix} \sigma_{11} & & \\ \sigma_{12} & \sigma_{22} & \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{Bmatrix}$$

Implied Covariance Matrix

Evaluating Covariance-Based Models

The log likelihood ratio, F_{ML} follows χ^2 distribution such that

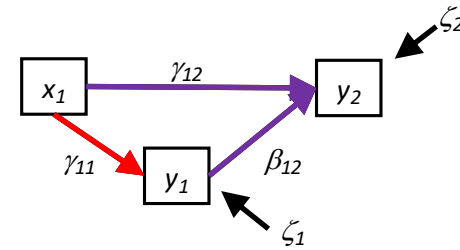
$$\chi^2 = (n-1)F_{ML}$$

- **Note scaling by sample size**
- **Large χ^2 implies LACK of fit**

Assumptions of Covariance Based Estimation

- Multivariate normality
 - Of residuals
 - Of data
- Linearity
- All correlations accounted for in model structure
- But... latent variables make magic possible

Comparison. Piecewise: Graphs to equations



$$y_1 = \gamma_{11}x_1 + \zeta_1$$

$$y_2 = \gamma_{12}x_1 + \beta_{12}y_1 + \zeta_2$$

Model fit evaluated via tests of conditional independence

Comparison. Traditional vs. piecewise SEM



Comparison. Traditional vs. piecewise SEM

| Variance-covariance | Piecewise |
|--|---|
| Single (global) variance-covariance matrix estimated | Variance-covariance matrices estimated for each endogenous variable |
| Simultaneous solution (computationally intensive) | Multiple solutions (modularized) |
| Fit to normal distribution | Incorporates various distributions (Poisson, Gamma, etc.) |
| Assumes independence | Can model non-independence (blocked, temporal, spatial, etc.) |
| Latent & composite variables | No latent or composite variables (yet*) |
| Non-recursive (cyclic) | Only for recursive (acyclic) |
| Multi-group models | Can estimate random components, but no formal χ^2 test |

Multiple Regression: Hidden Assumptions

equation form $y_1 = \gamma_{11}x_1 + \gamma_{21}x_2 + \zeta_1$

graphical form

- To estimate γ_{11} and γ_{21} you need to control for the unanalyzed correlation between x_1 and x_2 .
- In a factorial experiment, this correlation is 0!

Comparison. Graphs to equations

$y_1 = \gamma_{11}x_1 + \zeta_1$

$y_2 = \gamma_{12}x_1 + \beta_{12}y_1 + \zeta_2$

Comparison. Graphs to equations

equation form

$y_1 = \gamma_{11}x_1 + \zeta_1$

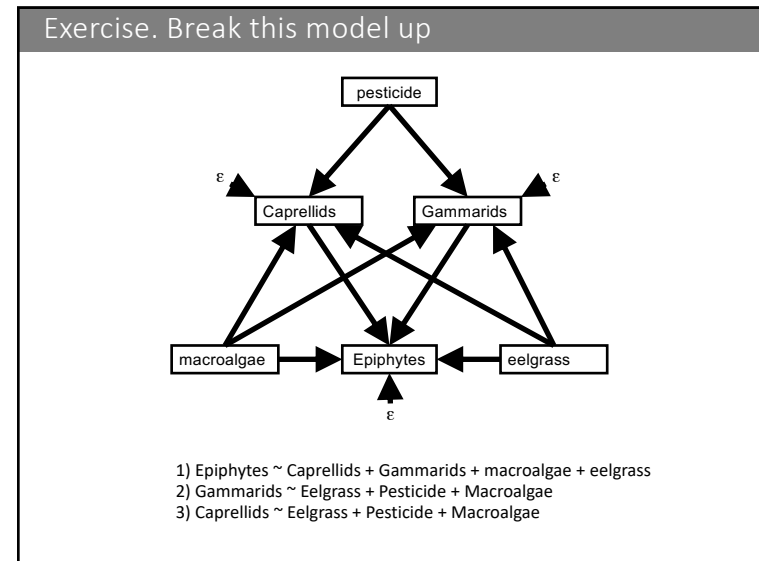
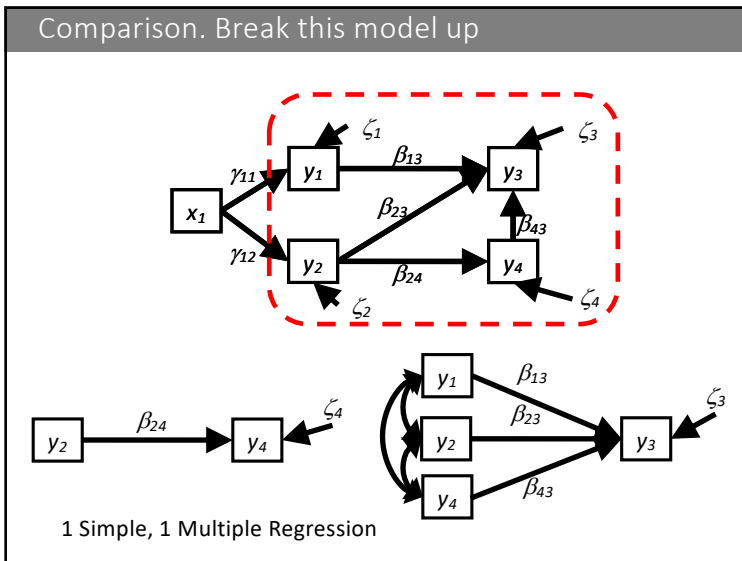
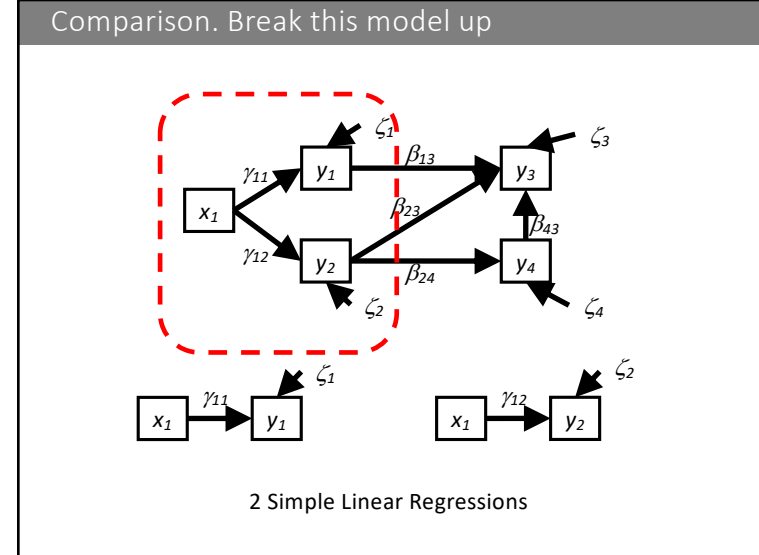
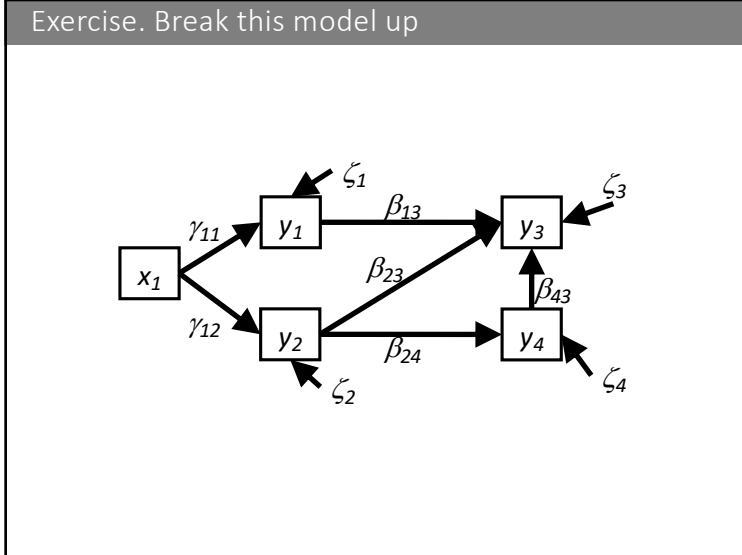
$y_2 = \gamma_{12}x_1 + \beta_{12}y_1 + \zeta_2$

Comparison. Graphs to equations

$y_1 = \gamma_{11}x_1 + \zeta_1$

$y_2 = \beta_{12}y_1 + \zeta_2$

$y_2 = \gamma_{12}x_1 + \zeta_2$



ACTIVITY

- Take your own causal diagram
- Break it up into the component models

Overview

1. Covariance vs. piecewise SEM
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3. Introduction to *piecewiseSEM*
4. Further adventures into *piecewiseSEM*

Directed Separation. Model fit

Does the model fit the data?

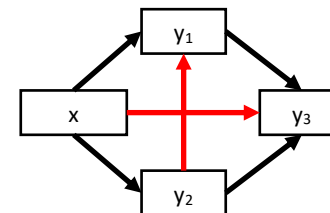
=

Does the model represent the data well?

=

Are we missing important information?

Directed Separation. Model fit



Did we get the topology right or are there unrecognized significant relationships?

Conditional Independence and Hard Causal Claims

```

    graph TD
      EC[Exogenous Cause] --> C[Cause]
      EC --> E[Effect]
      C --> M[Mediator]
      M --> E
      C --> E
  
```

- We assume that two variables not connected are independent, conditioned on their parent influences
- This is a HARD causal claim, setting a path to 0
- Testable

Mediator ⊥ Exogenous Cause | Cause

Directed Separation. D-sep

```

    graph TD
      X[X] --> Y1[Y1]
      X[X] --> Y2[Y2]
      Y1[Y1] --> Y3[Y3]
      Y2[Y2] --> Y3[Y3]
      X[X] -.- Y3[Y3]
  
```

- Concept from Graph Theory
- Two nodes are d-separated if they are *conditionally independent* e.g., the effect of x on y_3 is zero conditioning on the influences of y_1 and y_2

Directed Separation. Independence claims

The d-separation criterion for any pair of variables involves:

1. Controlling for common ancestors that could generate correlations between the pair
2. Controlling for causal connections through multi-link directed pathways via parents
3. Not controlling for common descendent variables.

Directed Separation. D-sep

```

    graph TD
      X[X] --> Y1[Y1]
      X[X] --> Y2[Y2]
      Y1[Y1] --> Y3[Y3]
      Y2[Y2] --> Y3[Y3]
      X[X] -.- Y3[Y3]
  
```

Test of directed separation:

1. Identify all independence claims
2. Evaluate each independence claim
3. Summarize information across all claims

Directed Separation. The Basis Set

Basis set = the smallest possible set of independence claims from a graph

- $x \perp y_3 \mid (y_1, y_2)$
- $y_1 \perp y_2 \mid (x)$

Directed Separation. Deriving the basis set

What is the basis set?

- $mass \perp dia \mid (canopy)$
- $mass \perp \# \mid (canopy)$
- $mass \perp \% \mid (canopy)$
- $dia \perp \# \mid (canopy)$
- $dia \perp \% \mid (canopy)$
- $\% \perp \# \mid (canopy)$

Directed Separation. Deriving the basis set

What is the basis set?

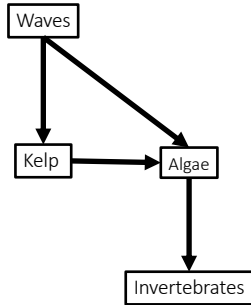
- $dia \perp \# \mid (canopy)$
- $dia \perp \% \mid (canopy, \#)$
- $canopy \perp mass \mid (dia)$
- $mass \perp \# \mid (dia, canopy)$
- $mass \perp \% \mid (dia, \#)$
- $canopy \perp \% \mid (\#)$

Directed Separation. Deriving the basis set

What is the basis set?

- $Pesticide \perp epiphytes$ (macroalgae, eelgrass, caprellids, gammarids)
- $Caprellids \perp gammarids$ (macroalgae, eelgrass, pesticide)

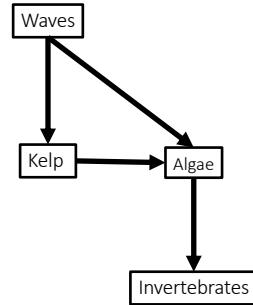
Check yourself with dagitty



```

forest_mod <- dagitty("dag{
waves -> kelp -> algae
algae -> inverts
waves -> algae
}")
  
```

Check yourself with dagitty



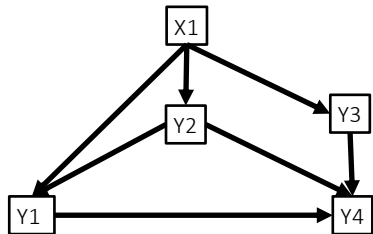
```

> impliedConditionalIndependencies(forest_mod)
  
```

```

inverts _||_ kelp | algae
inverts _||_ waves | algae
  
```

Exercise: What are your independence relationships?

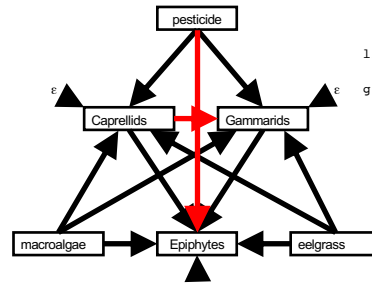


```

> impliedConditionalIndependencies(g)
x1 _||_ y4 | y1, y2, y3
y1 _||_ y3 | x1
y2 _||_ y3 | x1
  
```

Directed Separation. Deriving the basis set

What about Covariances?



```

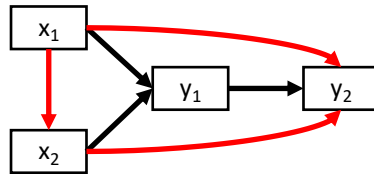
library(dagitty)
g <- dagitty("dag{
pesticide -> caprellids <- macroalgae
pesticide -> gammarids <- macroalgae
caprellids -> epiphytes <- gammarids
macroalgae -> epiphytes <- eelgrass
eelgrass -> caprellids
eelgrass -> gammarids
}")
  
```

```

> impliedConditionalIndependencies(g)
caprellids _||_ gammarids | eelgrass, macroalgae, pesticide
eelgrass _||_ macroalgae
eelgrass _||_ pesticide
epiphytes _||_ pesticide | caprellids, eelgrass, gammarids, macroalgae
macroalgae _||_ pesticide
  
```


Directed Separation. A note

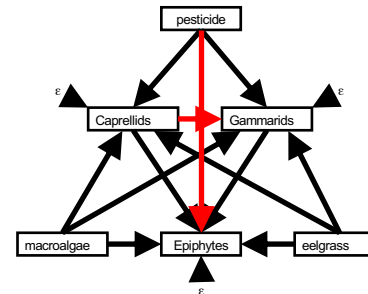
- Basis set generally excludes relationships among exogenous variables



1. $x_1 \perp y_2 \mid (y_1)$
2. $x_2 \perp y_2 \mid (y_1)$

Directed Separation. Deriving the basis set

Coding Covariances

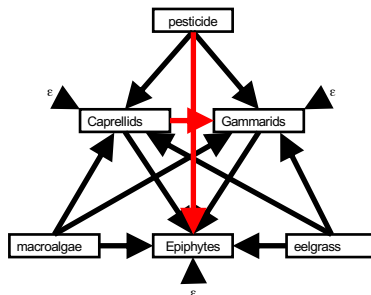


```
g2 <- dagitty("dag{
pesticide -> caprellids <- macroalgae
pesticide -> gammarids <- macroalgae
caprellids -> epiphytes <- gammarids
macroalgae -> epiphytes <- eelgrass
eelgrass -> caprellids
eelgrass -> gammarids
macroalgae <-> pesticide
eelgrass <-> macroalgae
eelgrass <-> pesticide
}")
```

```
> impliedConditionalIndependencies(g2)
```

```
caprellids _||_ gammarids | eelgrass, macroalgae, pesticide
eelgrass _||_ pesticide
```

Using lavaan shortcuts with dagitty

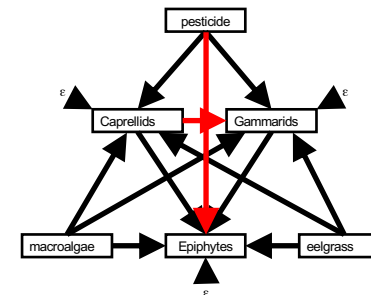


```
library(lavaan)
```

```
mod <- "
epiphytes ~ eelgrass + macroalgae + caprellids + gammarids
caprellids ~ eelgrass + macroalgae + pesticide
gammarids ~ eelgrass + macroalgae + pesticide
"
```

```
g3 <- lavaanToGraph(lavaanify(mod))
```

Using lavaan shortcuts with dagitty

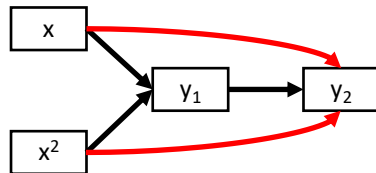


```
> impliedConditionalIndependencies(g3)
```

```
caprellids _||_ gammarids | eelgrass, macroalgae, pesticide
eelgrass _||_ pesticide
```

Directed Separation. Nonlinearities

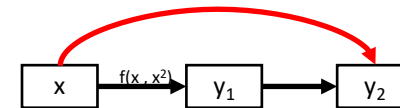
- Basis set generally excludes non-linear components (polynomials)



$$1. x \perp y_2 \mid (y_1)$$

Directed Separation. Nonlinearities

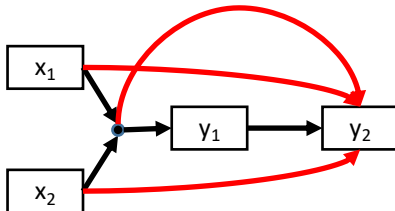
- Why? Paths are not inherently linear



$$1. x \perp y_2 \mid (y_1)$$

Directed Separation. Nonlinearities

- Basis set generally excludes non-linear components (interactions)

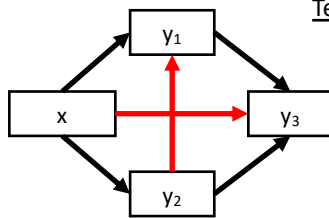


$$\begin{aligned} 1. & x_1 \perp y_2 \mid (y_1) \\ 2. & x_2 \perp y_2 \mid (y_1) \end{aligned}$$

ACTIVITY

- Take your causal diagram
- Derive the basis set
- Try it by hand before dagitty

Directed Separation. D-sep

Test of directed separation:

1. Identify all independence claims
2. **Evaluate each independence claim**
3. Summarize information across all claims

Directed Separation. Fisher's C

- Summarize independence claims across basis set:

$$C = -2 * \sum \ln(p_i)$$

p_i = the P-values of all tests of conditional independence

- C has a χ^2 -square distribution with $2k$ degrees of freedom
- k = # of elements of the basis set

Directed Separation. Fisher's C

What is $p < 0.05$?

- You are likely missing some associations
- You *reject* this model
- The way forward: adding links or different model structure? (look at d-sep tests)
- To re-iterate, $p \geq 0.05$ is GOOD

Directed Separation. Model selection

- Fisher's C can be used to construct model AIC:

$$AIC = C + 2K$$

- K = # of likelihood parameters estimated (not to be confused with k)
- Can be extended to small sample size:

$$AICc = C + 2K(n / (n - K - 1))$$

Directed Separation. Complexity and sample size

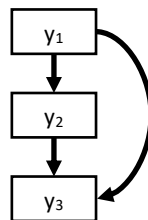
- The t -rule applies in generating model d.f. for the C statistic
- Shipley suggests need only enough individual d.f. to fit each component model
 - This is OK, until you try and do prediction
- Or, d -rule (Grace et al 2015):
 - $d = \#$ of samples / $\#$ of pathways
 - $d \geq 5$

Directed Separation. Sample size

- More is always better...
 - Low sample size leads to non-significant d-sep tests
 - Low sample size also leads to non-significant path coefficients
 - End up with a 'good fitting' model that says *nothing*

Directed Separation. Saturated models

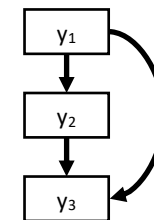
What is the basis set?



- There is no basis set for a saturated model (all paths are represented)
- No d-sep tests, so, AIC = 2K

Directed Separation. Saturated models

What is the basis set?



Options:

1. Remove the mediating variable y_2 and test submodel
2. Rely on other indicators of model fit (e.g., R^2)

A global test is not the be all-end all of models

Overview

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piecewiseSEM.

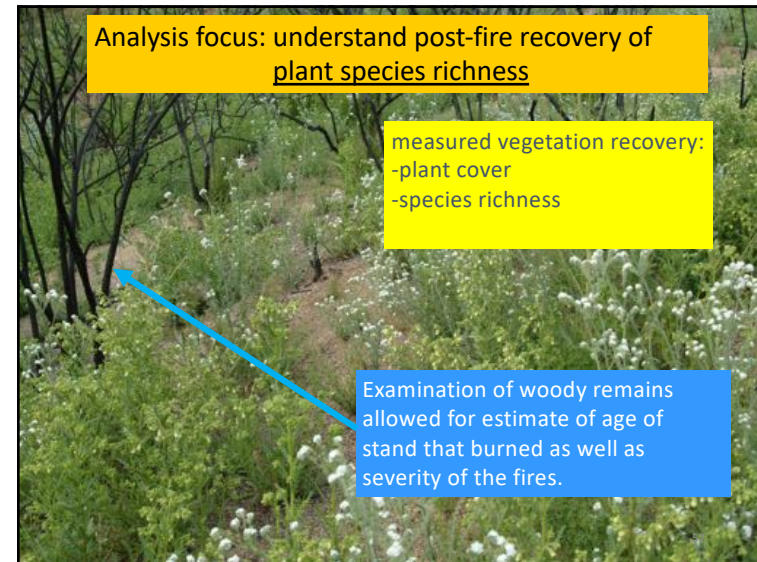
piecewiseSEM: Piecewise structural equation modeling in R for ecology, evolution, and systematics

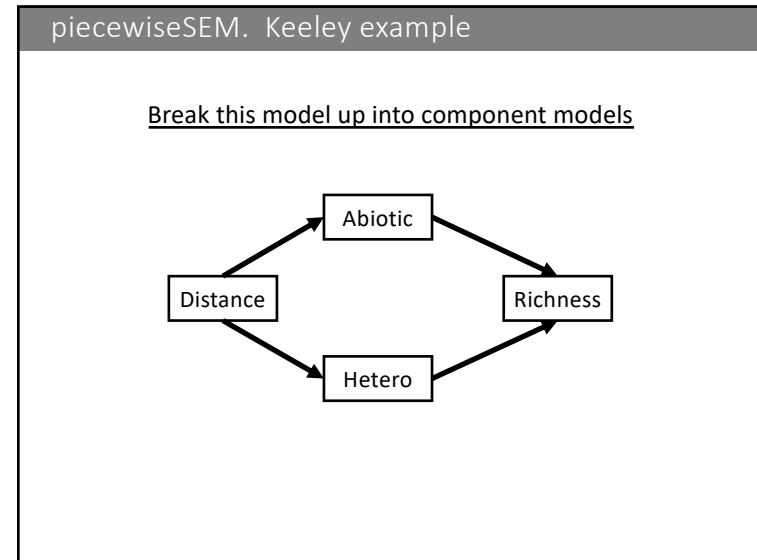
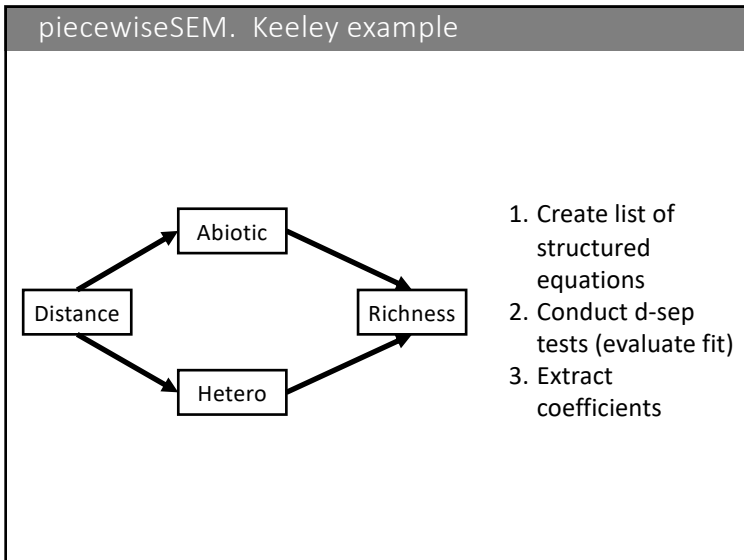
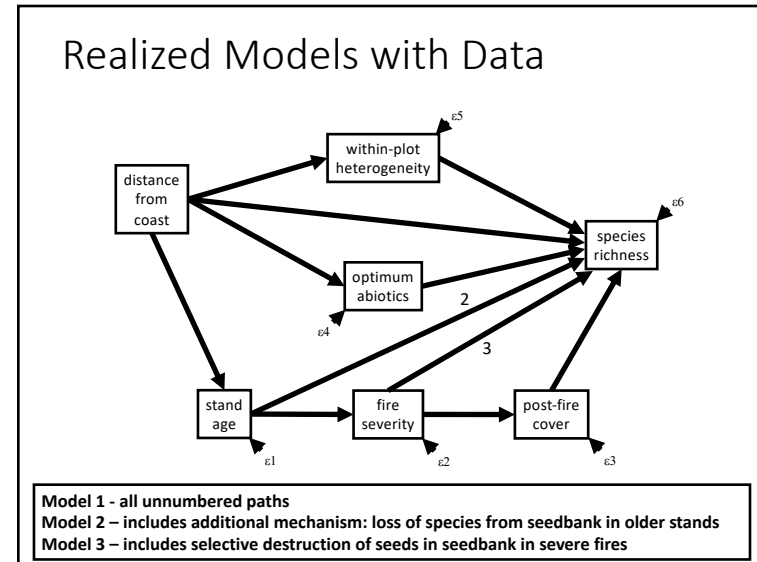
```
install.packages("devtools")
library(devtools)
install_github("jstefche/piecewiseSEM@devel")
library(piecewiseSEM)
```

Mediation in Analysis of Post-Fire Recovery of Plant Communities in California Shrublands

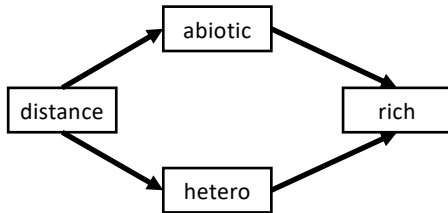


Analysis focus: understand post-fire recovery of plant species richness





piecewiseSEM. Store list of equations



```

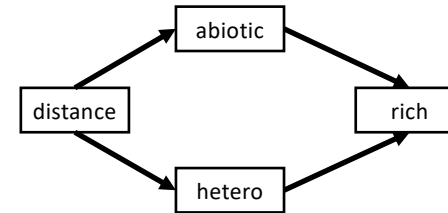
# Read in data
keeley <- read.csv("../data/keeley_rawdata_select4.csv")

### Fit Individual Relationships

abiotic_mod <- lm(abiotic ~ distance, data = keeley)
hetero_mod <- lm(hetero ~ distance, data = keeley)
rich_mod <- lm(rich ~ abiotic + hetero, data = keeley)

```

piecewiseSEM. Store list of equations



```

##Fit the model

keeley.sem <- psem(
  abiotic_mod,
  hetero_mod,
  rich_mod,
  keeley
)

```

piecewiseSEM. Store list of equations

```

keeley.sem

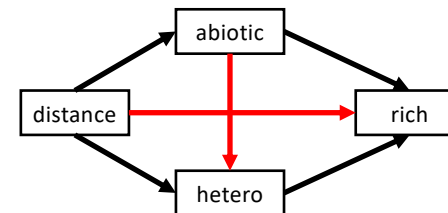
Structural Equations:
lm: abiotic ~ distance
lm: hetero ~ distance
lm: rich ~ abiotic + hetero

Data:
  distance elev abiotic age hetero firesev cover rich
1 53.40900 1225 60.67103 40 0.757065 3.50 1.0387974 51
2 37.03745 60 40.94291 25 0.491340 4.05 0.4775924 31
3 53.69565 200 50.98805 15 0.844485 2.60 0.9489357 71
4 53.69565 200 61.15633 15 0.690847 2.90 1.1949002 64
5 51.95985 970 46.66807 23 0.545628 4.30 1.2981890 68
6 51.95985 970 39.82357 24 0.652895 4.00 1.1734866 34
...with 84 more rows

[1] "class(psem)"

```

piecewiseSEM. D-sep tests



```

# Get the basis set
basisSet(keeley.sem)

$`1`
[1] "distance" "rich" "abiotic" "hetero"

$`2`
[1] "abiotic" "hetero" "distance"

```

piecewiseSEM. D-sep tests

```

claim1 <- lm(rich ~ distance + abiotic + hetero, keeley)
coefs(claim1)

```

| Response | Predictor | Estimate | Std.Error | DF | Crit.Value | P-Value | Std.Estimate |
|----------|-----------|----------|-----------|----|------------|---------|--------------|
| 1 rich | distance | 0.6404 | 0.1565 | 86 | 8.0933 | 0.0001 | 0.3743 *** |
| 2 rich | abiotic | 0.5233 | 0.1756 | 86 | 2.9793 | 0.0038 | 0.2660 ** |
| 3 rich | hetero | 33.4010 | 11.1187 | 86 | 3.0040 | 0.0035 | 0.2539 ** |

piecewiseSEM. D-sep tests

```

claim2 <- lm(hetero ~ abiotic + distance, keeley)
coefs(claim2)

```

| Response | Predictor | Estimate | Std.Error | DF | Crit.Value | P-Value | Std.Estimate |
|----------|-----------|----------|-----------|----|------------|---------|--------------|
| 1 hetero | abiotic | 0.0022 | 0.0017 | 87 | 1.3296 | 0.1871 | 0.1491 |
| 2 hetero | distance | 0.0036 | 0.0015 | 87 | 2.4742 | 0.0153 | 0.2774 * |

piecewiseSEM. D-sep tests

```

# Compute Fisher's C & compare to Chi-square distribution
C <- -2 * (log(coefs(claim1)[1, 7]) + log(coefs(claim2)[1, 7]))
1 - pchisq(C, 2 * 2)

```

[1] 0.0002223955

piecewiseSEM. D-sep tests

```

> dSep(keeley.sem)

```

| Independ.Claim | Estimate | Std.Error | DF | Crit.Value | P-Value |
|--------------------------|-------------|-------------|----|------------|------------------|
| 1 rich ~ distance + ... | 0.640431807 | 0.156457462 | 86 | 8.093329 | 9.564005e-05 *** |
| 2 hetero ~ abiotic + ... | 0.002229248 | 0.001676649 | 87 | 1.329585 | 1.871306e-01 |

piecewiseSEM. D-sep tests

```

> dSep(keeley.sem, conditioning= TRUE)

      Independ.Claim  Estimate  Std.Error DF  Crit.Value  P.Value
1 rich ~ distance + abiotic + hetero  0.640431807  0.156457462  86   8.093329  9.564005e-05 ***
2 hetero ~ abiotic + distance  0.002229248  0.001676649  87   1.329585  1.871306e-01
    
```

piecewiseSEM. Re-assess fit

```

# Add direct effect path back into model
keeley.sem2 <- update(keeley.sem, rich ~ abiotic + hetero + distance)
    
```

piecewiseSEM. Re-assess fit

```

dSep(keeley.sem2)

      Independ.Claim  Estimate  Std.Error DF  Crit.Value  P.Value
1 hetero ~ abiotic + ...  0.002229248  0.001676649  87   1.329585  0.1871306

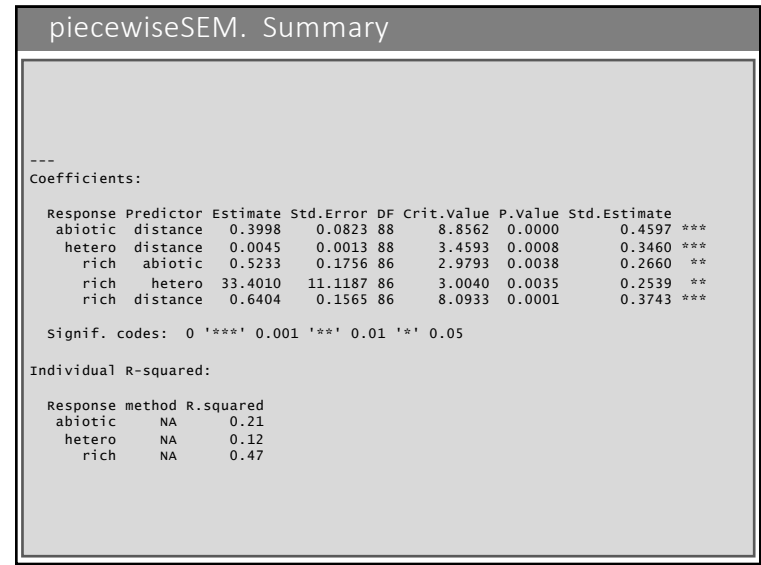
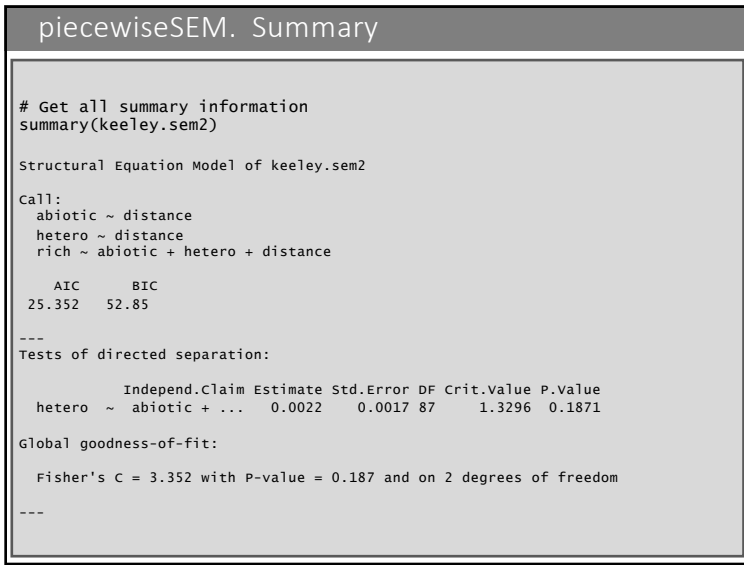
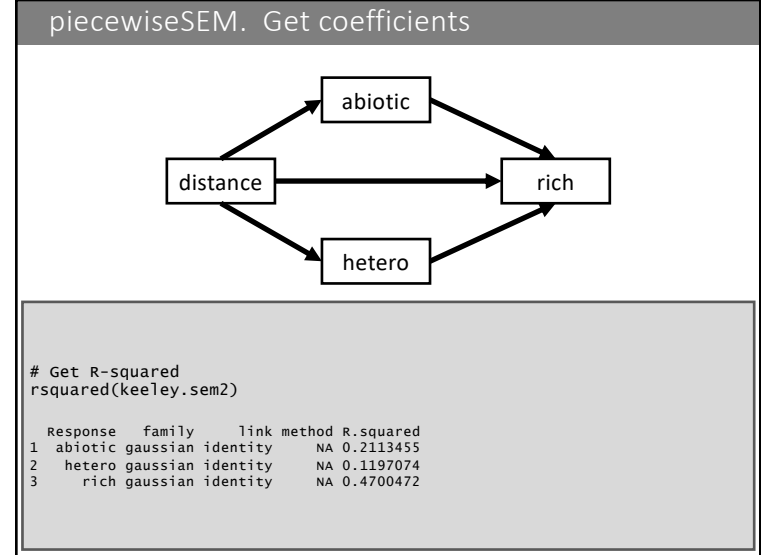
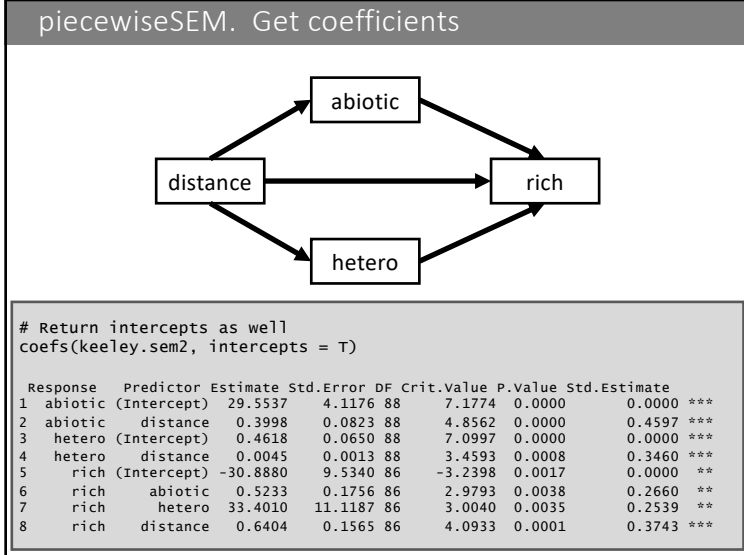
fisherC(keeley.sem2)
      Fisher.C df P.Value
1      3.352  2  0.187
    
```

piecewiseSEM. Get coefficients

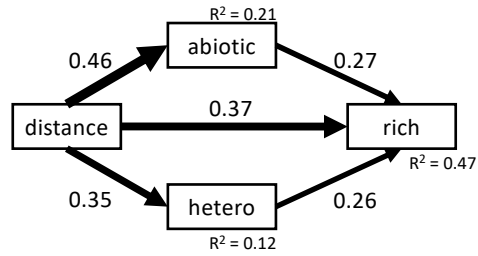
```

# Get coefficients
coefs(keeley.sem2)

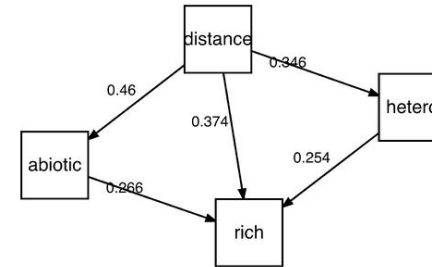
      Response Predictor  Estimate  Std.Error DF  Crit.Value  P.Value  Std.Estimate
1  abiotic distance    0.3998    0.0823  88   8.8562  0.0000    0.4597 ***
2  hetero distance     0.0045    0.0013  88   3.4593  0.0008    0.3460 ***
3   rich abiotic       0.5233    0.1756  86   2.9793  0.0038    0.2660 **
4   rich hetero       33.4010   11.1187  86   3.0040  0.0035    0.2539 **
5   rich distance     0.6404    0.1565  86   8.0933  0.0001    0.3743 ***
    
```



piecewiseSEM. Summary

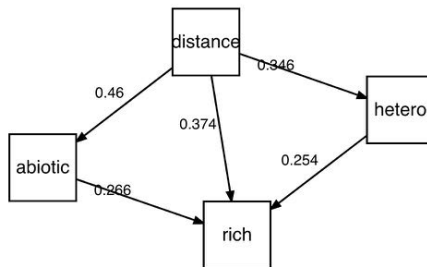


piecewiseSEM. plot



```
#plot the model
plot(keeley.sem2)
```

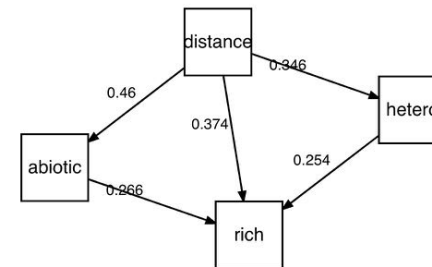
piecewiseSEM. Making a better plot



```
#make a better plot
keeley.plot2 <- plot(keeley.sem2, return=TRUE)

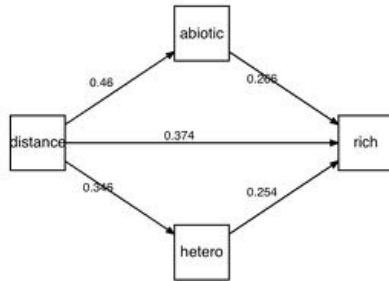
#what's in there?
get_node_df(keeley.plot2)
```

piecewiseSEM. Making a better plot



```
> get_node_df(keeley.plot2)
  id type label nodes shape color fillcolor
1 1 lower abiotic abiotic rectangle black white
2 2 lower hetero hetero rectangle black white
3 3 lower rich rich rectangle black white
4 4 lower distance distance rectangle black white
```

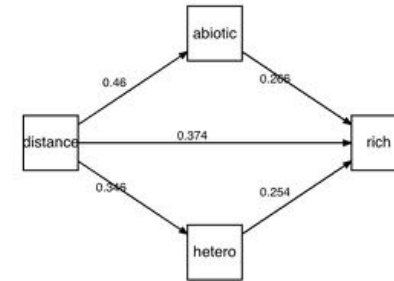
piecewiseSEM. Making a better plot



```

keeley.plot2 %>%
  set_node_attrs(node_attr = x, values = c(2.5,2.5,4,1)) %>%
  set_node_attrs(node_attr = y, values = c(3,1,2,2)) %>%
  render_graph()
  
```

piecewiseSEM. Making a better plot

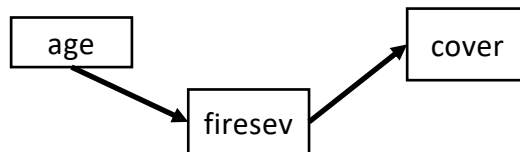


```

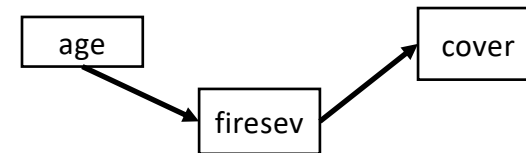
plot(keeley.sem2,
  node_attrs = list(x = c(2.5,2.5,4,1),
    y = c(3,1,2,2),
    shape = "rectangle",
    fillcolor = "white"))
  
```

piecewiseSEM. Exercise

1. Fit this model!
2. Fill in Standardized Coefficients and R² for this model
3. Assess any independence claims
4. Calculate summed indirect effects of age on cover



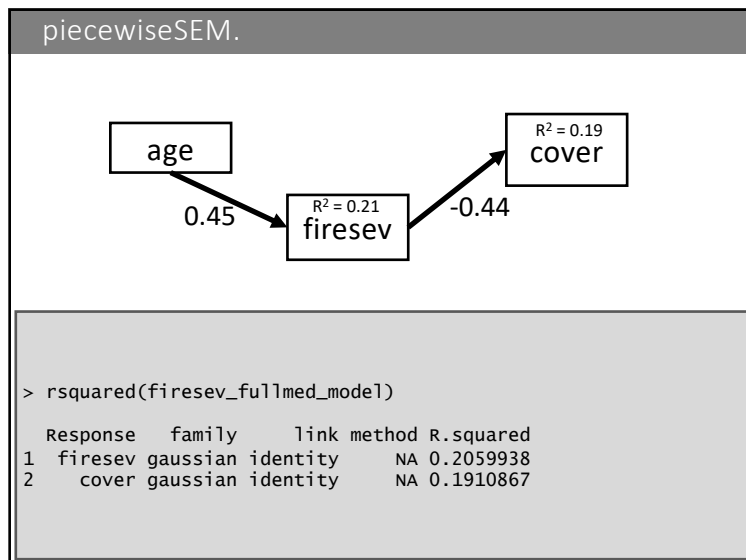
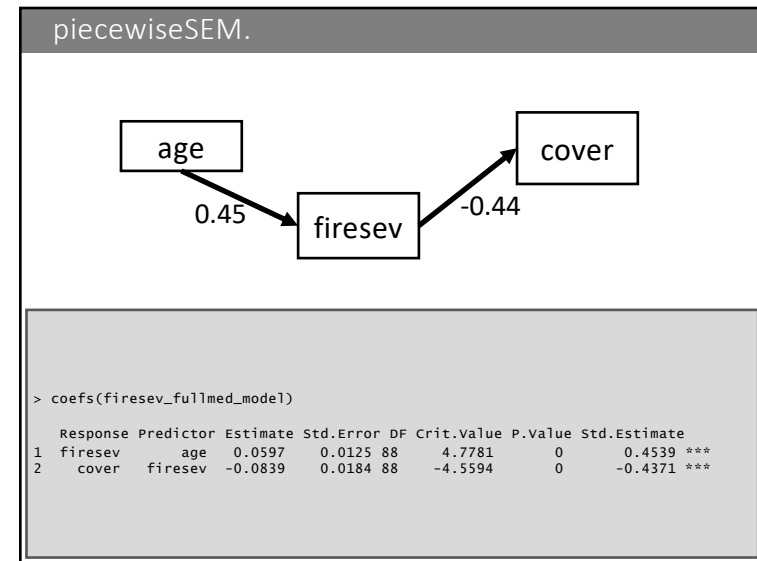
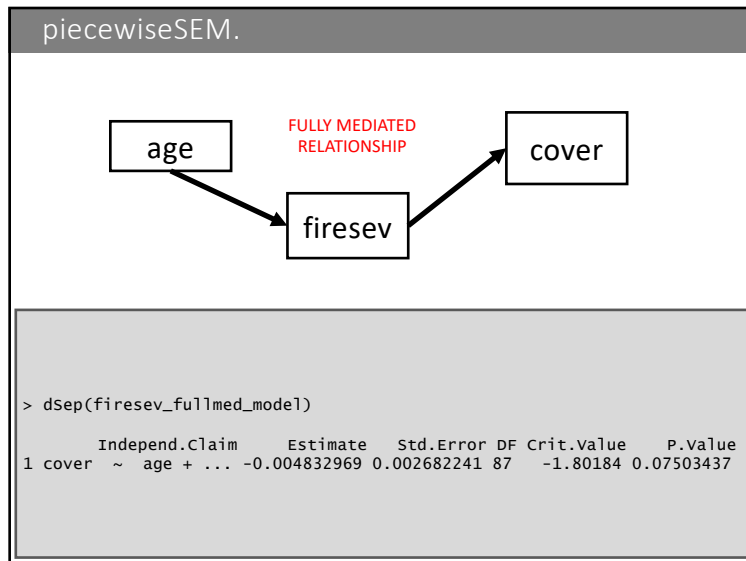
piecewiseSEM.



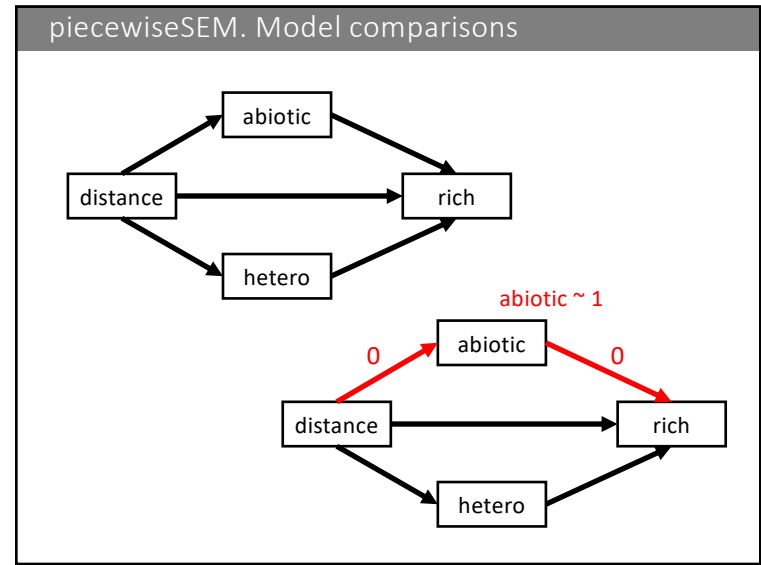
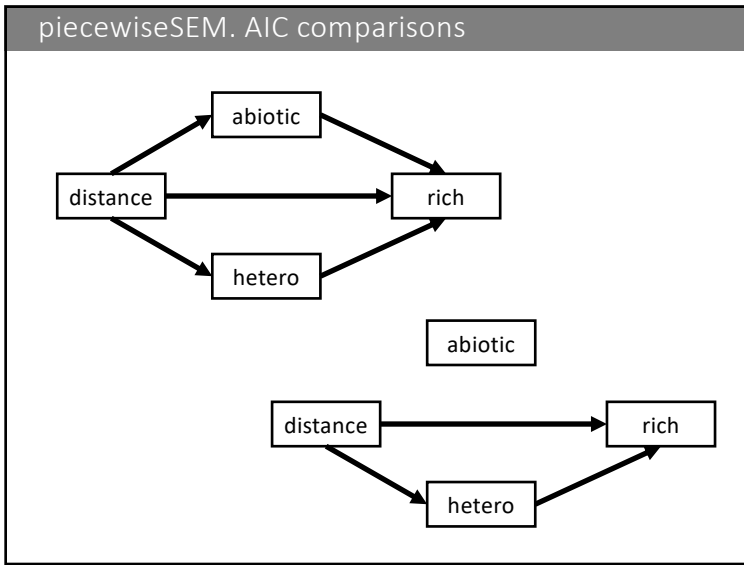
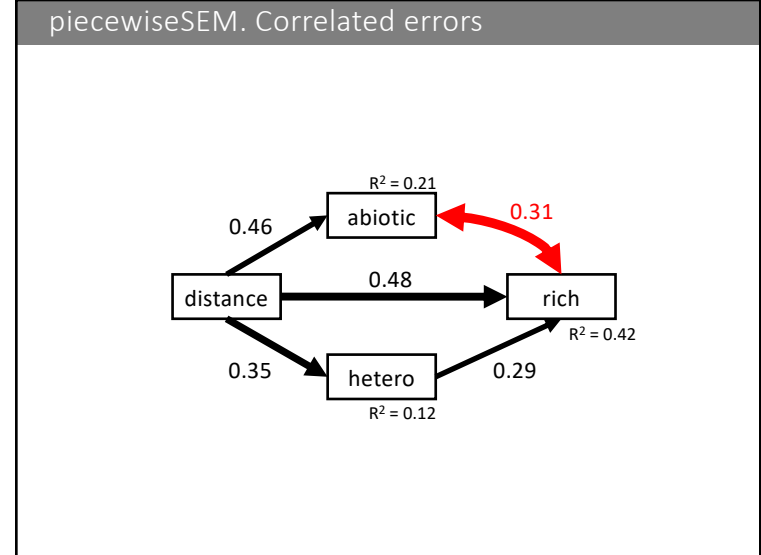
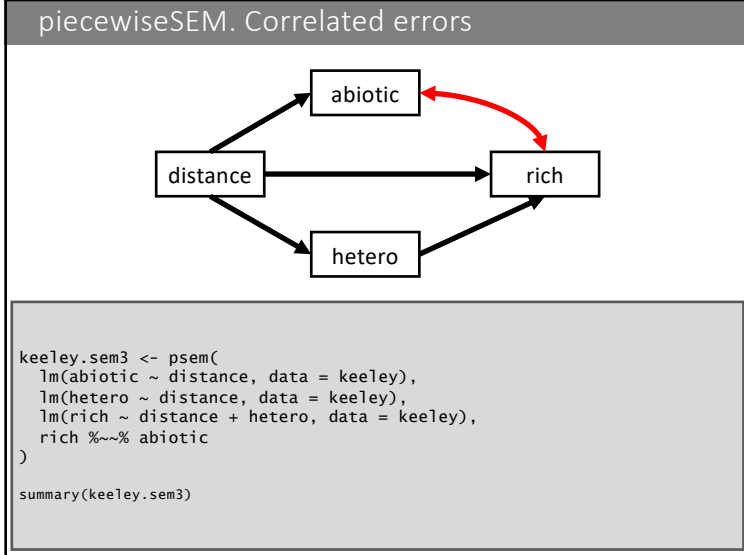
```

mod_firesev <- lm(firesev ~ age, data=keeley)
mod_cover <- lm(cover ~ firesev, data=keeley)

firesev_fullmed_model <- psem(
  mod_firesev,
  mod_cover,
  keeley
)
  
```



- Overview
1. Covariance vs. piecewise SEM
 2. Assessing Model Fit: Tests of directed separation
 3. Introduction to *piecwiseSEM*
 4. Further adventures into *piecwiseSEM*



piecwiseSEM. Fit new model

```

keeley.sem4 <- psem(
  lm(hetero ~ distance, data = keeley),
  lm(rich ~ distance + hetero, data = keeley),
  abiotic ~ 1,
)

```

piecwiseSEM. Compare with LRT

```

> anova(keeley.sem2, keeley.sem4)
Chi square difference test

      AIC   BIC Fisher.C Fisher.C.Diff DF.diff P.value
1  25.352 52.850   3.352             11.17     2 0.0038 **
vs 2  28.522 46.021  14.522

```

piecwiseSEM. Compare with AIC

```

AIC(keeley.sem2)[1]
[1] 25.352
AIC(keeley.sem4)[1]
[1] 28.522

```

piecwiseSEM. AIC comparisons

piecewiseSEM. Understanding Relationships

3 Ways to View the independent effect of distance on richness:

1. Partial Regression Plots
 - residual of richness + distance effect \sim residual of distance after accounting for abiotic + hetero
 - VERY VERY conservative, but, accurate
2. Component Residual Plots
 - residual of richness + distance effect \sim distance
3. Fit of richness \sim distance holding other variables at median
 - Preferred, as in sensible units

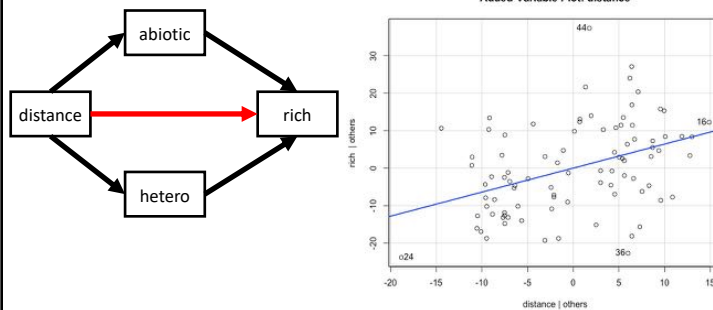
piecewiseSEM. Digging into psem

```
#psem objects are lists of models
> keeley.sem2[[3]]
```

```
Call:
lm(formula = rich ~ abiotic + hetero +
distance, data = keeley)
```

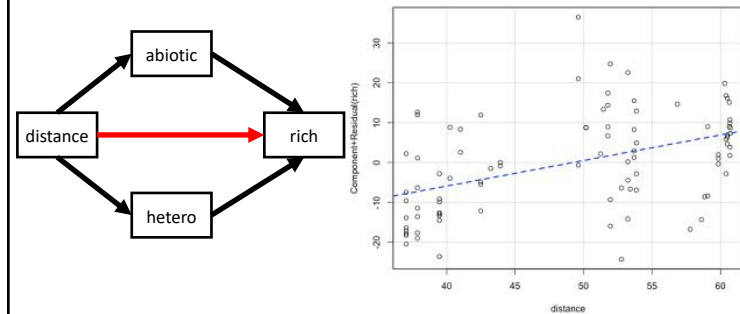
```
Coefficients:
(Intercept)      abiotic      hetero
distance
   -30.8880      0.5233      33.4010
   0.6404
```

piecewiseSEM. Partial correlations



```
#Added Variable Plots
library(car)
avPlot(keeley.sem2[[3]], variable = "distance")
```

piecewiseSEM. Component + Residual



```
#CR Plot
crPlot(keeley.sem2[[3]], variable = "distance")
```


piecewiseSEM. Distance Effect When All Others are At Median

```

#visreg
library(visreg)

visreg(keeley.sem2[[3]],
       xvar = "distance")
    
```

piecewiseSEM. Working with Controlled Data

```

> vr <- visreg(keeley.sem2[[3]],
              xvar = "distance")

> head(vr$res)
  abiotic hetero distance rich visregRes visregPos
1 48.03605 0.684314 53.40900 50 41.95826 FALSE
2 48.03605 0.684314 37.03745 50 41.15731 TRUE
3 48.03605 0.684314 53.69565 50 64.10536 TRUE
4 48.03605 0.684314 53.69565 50 56.91605 TRUE
5 48.03605 0.684314 51.95985 50 73.34811 TRUE
6 48.03605 0.684314 51.95985 50 39.34695 FALSE
    
```

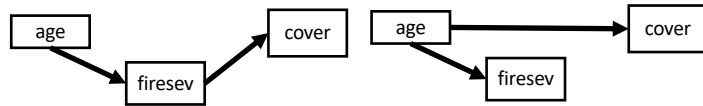
Visualizing While Controlling Covariates

- Useful for displaying trends, particularly with complex models where bivariate correlations are messy
- Can be used for any multiple regression (single model or list)
- Not applicable to simple regression ($Y \sim X$)

piecewiseSEM. Exercise

1. Fit and compare these models!
2. Visualize the effect of fire severity's effect on cover given this model structure

piecewiseSEM. Exercise

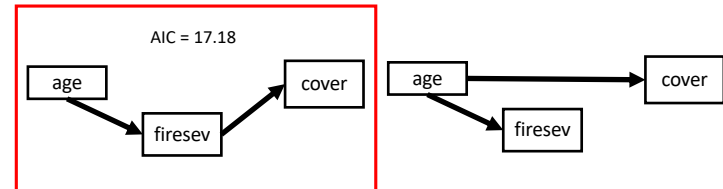


```

fullMed <- psem(
  lm(cover ~ firesev, data = keeley),
  lm(firesev ~ age, data = keeley),
  keeley
)

noMed <- psem(
  lm(firesev ~ age, data = keeley),
  lm(cover ~ age, data = keeley),
  data = keeley)
  
```

piecewiseSEM. Exercise



```

> anova(fullMed, noMed)
Chi square difference test

```

| | AIC | BIC | Fisher.C | Fisher.C.Diff | DF.diff | P.value |
|------|--------|--------|----------|---------------|---------|---------|
| 1 | 17.180 | 32.179 | 5.180 | | | |
| vs 2 | 25.116 | 40.115 | 13.116 | 7.936 | 0 | 0 *** |

piecewiseSEM. Exercise

```

partialMed <- psem(
  lm(firesev ~ age, data = keeley),
  lm(cover ~ age + firesev, data = keeley),
  data = keeley)

visreg(partialMed[[2]], xvar = "firesev")
  
```

